Practice Problem Set 4

Question 1 We analyze a contract between an employer (the Principal) and an employee (the Agent) in a situation of moral hazard. The agent has two possible effort levels, e^H and e^L with associated costs $c^H = 1$ and $c^L = 0$. The reservation utility of the agent is equal to $\overline{U} = 1$. The principal is risk-neutral and the agent is risk averse with a utility function $u(w) = \sqrt{w}$. There are two possible outputs, $x^L = 2$ and $x^H = 10$. The probabilities of each output as a function of the agent's effort are given by the following table.

	x^L	x^H
e^L	1	0
e^H	$\frac{1}{2}$	$\frac{1}{2}$

(a) First suppose that the principal can observe the agent's effort and chooses a wage w^H for the high effort level e^H and w^L for the low effort level e^L.

(i) Write down the expected profit of the principal when the agent chooses e^H, Π^H and when he chooses e^L, Π^L.

(ii) Write down the participation constraints of the agent when he chooses effort e^H and when he chooses effort e^L.

(iii) What are the wages in the optimal contract? What is the principal's expected profit when he implements the high and low effort levels? Show that the principal prefers to implement the high effort level e^H.

(b) We now suppose that the principal cannot observe the effort level and bases the salary on the realized output level. She pays a salary w^H for output x^H and a wage w^L for output x^L. We first suppose that the principal wants to implement the high level of effort e^H.

(i) Write down the profit of the principal.

(ii) Write down the participation constraint of the agent when he chooses effort level e^H.

(iii) Write down the incentive constraint of the agent.

(iv) Graph the participation and incentive constraint of the agent, as well as the isoprofit curves, in a space (t^H, t^L) where $t^H = \sqrt{w^H}, t^L = \sqrt{w^L}$.

(v) Deduce that at the optimal contract, we must have $w^H = 9, w^L = 1$. Compute the expected profit of the principal.

(c) We next suppose that the principal's objective is to implement effort level e^L.

(i) Write down the expected profit of the principal.

(ii) Write down the participation constraint of the agent when he chooses effort level e^L.

(iii) Deduce the optimal salary and the expected profit of the principal. Check that the principal still prefers to implement effort level e^H.